A K-SHORTEST PATH ALGORITHM FOR TRANSIT NETWORK BASED ON PARTIAL OVERLAP
Zhuo SUN¹), Hirokazu KATO²) and Yoshitsugu HAYASHI³)

1) PhD Student, Graduate School of Environmental Studies, Nagoya University
2) Associate Professor, Graduate School of Environmental Studies, Nagoya University
3) Professor and Dean, Graduate School of Environmental Studies, Nagoya University

Abstract: The study discusses seamlessly integrating multiple modes of transit in GIS and calculating the k-shortest paths in the network between origin and destination with a new topological structure and a new algorithm based on partial overlap. A node projecting method and two dummy links have been used to connect different networks. An efficient and loopless k-shortest path algorithm based on this topological structure is introduced. The result paths can be offered visually through GIS. Users can get very precise and efficient travel information. The partial overlap indicator introduced in this study could also be used to evaluate a park and ride system and scatter the park and ride sites to alleviate the congestion.

1. Introduction

The transit system in a city is a very important transport method for reducing traffic demands, traffic congestion and energy consumption, thereby alleviating air pollution and CO2 emission. However, it is not always easy for people to use the transit system around a metropolis. The problem is not about availability of services, but about finding a travel plan for a trip. Finding travel plans for non-recurrent trips from a wide selection and combination of bus, subway, and train routes is not an easy job even for local people, let alone tourists. Demands for non-recurrent trips may occur when there are special events and when services for the commuters' customary travel plans are interrupted or detoured under unexpected situations.

A transit advanced traveler information system (TATIS) can meet the needs mentioned above, it is essential for TATIS to offer travelers the accurate and reliable transit information and let them know the actual operation situation, such as the needed time/cost, the transfer times and transfer sites. The advanced traveler information system could provide users the best travel plan consisting of a path with transfers that minimize the cost of the whole path. Sometimes users also want to know other alternative paths beside the best one. The reason is there are many uncertainties in the real world and the system should provide more choices to users to
help them decide for themselves. To meet it, k-shortest path algorithms have been used. Yen (1971) first introduced a k-shortest path searching method by deleting nodes from the network. Since, then several k-shortest algorithms have been suggested. Although a k-shortest path algorithm can provide several alternative paths, it can be limited by the heavy overlap of links amongst derived paths. This may lead to wrong travel information being provided to the users. Barra et al. (1993) proposed a link penalty method such that the network is modified by increasing the cost of all links on the shortest path. Yongtaek(2005) introduce a modified method of Barra that measures the overlap of the k-shortest paths as a similarity criterion.

The objective of this study is to develop a GIS based k-shortest path generating system on the transit network as a part of TATIS. A new path penalty method named partial overlap has been developed so that only part of the links in the shortest path could be updated. It can provide users accurate paths on a transit network when given an origin and destination.

Most existing TATIS have three disadvantages, namely 1) the resulting paths are not very accurate. It means that the selected paths do not coincide with the real world because some factors are neglected when merging different transit networks. 2) Algorithm and database structure is quite complicated. It cannot calculate all feasible paths and paths with more than 3 transfers are neglected. 3) Path similarity and transfer similarity exist in the result paths because many of the alternative paths derived from the k-shortest path algorithm are likely to share lots of links, so they could not represent heterogeneity.

In order to take the advantages and overcome the disadvantages described above, in this study the road network and transit lines are integrated seamlessly into a comprehensive network. All factors that affect the travel plan are stored in the attributes table of the integrated network. This can greatly simplify the data structure and make it direct to be accessed so that the standard shortest path algorithm does not need any changes. To provide alternative paths other than the best one a k-shortest path algorithm is applied. After this part every step will be described in detail. First, the network structure will be discussed, and then the new k-shortest path algorithm will be introduced. Then a case which can help understand the algorithm will be shown. Finally the conclusion discusses the advantages and the disadvantages of this approach.

2. NETWORK STRUCTURE
2.1 Construction of Networks in GIS

First, road layer is created based on raster image, above which the topological structure of road network is built. The attributes of road links is calculated by using spatial function of GIS and stored in the road network directly. The item of Length can be gained through the length of the vector “polyline” in GIS and then divided by the scale of the raster image. The length unit is set to kilometer. The item of Travel Time can be gained through the average walking speed (5km/hour) and the length.

The transit network is stored on an independent layer over the road layer. Transit lines can be created by referring to roads lines. Then stops can be built on the lines according to their real location, and then the lines are split by the created stops. Finally connect lines segments and stops to form a transit network.

The data structure of links in the transit network is similar to the links in road network except for the item of Travel Time. It can be calculated according to the real road property and the transit mode. For example, there is a bus route in the network. The route’s Travel Time can be obtained based on bus speed (about 20km/hour as usual) and the route length. Therefore, the value of Travel Time in a bus link is one sixth of the overlaid road link. If the ticket price increases along with the increasing of mileage during transit, the ticket price should be converted to equivalent time by using an appropriate formula. This issue will be discussed later.

2.2 The Combination of Road and Transit Networks under GIS

After road network and transit network are built, they need to be integrated in order to perform further calculation and analysis. The popular methods that integrate stop nodes in the transit network and the nearest intersection nodes in the road network are: first, from every stop node on the route layer find the nearest intersection node on the road layer; and then the ID of stop nodes is substituted by that of intersection nodes. This method of integration is very simple by using spatial functions on GIS. But it still has some disadvantages.

The topological structure proposed in this study uses two dummy links to link every stop in transit network to road network. The stops are projected to road network with accurate geometrical information. In this way, integrated network shows a very simple and accurate structure to be calculated and modified. Network impedances, such as cost, distance, transfer,
waiting time and traveling time could be directly stored on corresponding links and existing shortest and k-shortest path algorithms can be applied without any modification.

There are some points that should be noticed when forming the topological structure:

1. If many transit routes have their stops at the same location of a road, these stops cannot be merged into one node. Each link should be split by its own stop node. Therefore, the transit route network is formed by many connected links that are parallel to each other. (Distances are 0)

2. All routes in the transit route network and all roads in the road network should be converted to one-way link to match the "bigraph" topology. In practice, a two-way road or route can be presented by two directed links which are parallel and have opposite direction. After that the first principle also should be considered for setting up stops on each link.

3. **ALGORITHM OF K-SORTEST PATH**

To avoid the path similarity and transfer similarity, this study introduces a new k-shortest path algorithm based on partial overlap which extends the path overlap algorithm to the new structure proposed in this study. In the partial overlap procedure, only selected links will be overlapped, not all of them. This can efficiently find more dissimilar paths over the transit network and especially more dissimilar transfers over the transit network. Users can also define which mode of transit they want to alternate and the system will provide more alternative paths with more changes in this mode.

3.1 **Data Structure and Weights**

It is necessary to build the data structure in computer memory before inquiry. Since the final merged structure of this research is only a simple bigraph, the process of building data structure becomes very simple. Just reading the nodes and links in the road network and
transit one and inserting them into a list in computer memory by order can make it. The users can determine the weights of various links with user interface before the program reads link impedance from the network to the memory. The final impedance values are equal to the value of Travel Time multiplied by the weight. For example, users want to walk on less distance, reduce the time of getting on and off and reduce the number of transfers due to his heavy luggage. He can set up a higher weight for the link of road network and dummy link. Thus the higher values could be gotten through the value that Travel Time multiplied by weight inputted. Then user will get a result path in which there are smaller walking distance and fewer transfers.

3.2 Applying the K-shortest Path Algorithm

Existing researches did not completely combine road and transit networks but use a modified version of Shortest Path Algorithm along with Route Table and Connectivity Matrix. Thus the process of generating an optimal path is very complicated. The data structure produced in our study is just a bi-graph; therefore, any standard Shortest Path Algorithm and K-Shortest Path Algorithm can be applied directly without any adjustment. Here, a k-shortest path algorithm which uses partial link penalty method has been adopted in this topological structure. The algorithm builds new path based on the degree of partial overlapping between each path and travel cost, and stops building when the degree of overlapping ratio exceeds its criterion. First some variables are denoted as follows:

- $l_{rs}^n$: length of n-th path for origin-destination pair rs
- $p_{rs}^n$: n-th path for origin-destination pair rs
- $P_{rs}^n$: path set for origin-destination pair rs; $P_{rs}^n = \{p_{rs}^1, l_{rs}^1\}$
- $OL_{rs}^n$: partial overlapping length of n-th path to the length of k-th path for origin-destination pair rs
- $OP_{rs}^{nk}$: degree of partial overlap between the length of n-th path and the length of k-th path for origin-destination pair rs; $OP_{rs}^{nk} = \frac{OL_{rs}^n}{P_{rs}^k}$
- $OP$: maximum degree of partial overlap
- $QL$: link penalty; $QL = \left(\frac{QL}{Q_L}\right)^\alpha$
- $\alpha$: positive parameter

Then the procedure of calculation can be described as follows:
Step 0: Set Op and n=1
Step 1: Calculate the shortest path in current network, add $p_n^T, p_n^P$ to path set
Step 2: Set selected link cost in network = link cost * Oz
Step 3: Set n=n+1, $O_{p_{k/n}} = \frac{O_{p_{k/n}}}{n}$
Step 4: if $O_{p_{k/n}} > O_p$ then stop, otherwise go to step 1

The core of the procedure is selecting the links to be updated and calculate the partial overlap. From users’ point of view, when they want to find out some different travel plans in the transit network, they prefer to get some paths that have less similarity in transfers. To realize this purpose this study chooses the transit links as the parameter to be updated and to be used to calculate the partial overlap.

One big problem of this algorithm is how to choose Op, i.e., the degree of partial overlap. Making this quantity too small would result in the user being provided needlessly expensive paths. If it is too large then of course one runs into the overlap problem. How should one choose Op? A reasonable solution is: first, set a random value of Op. Then distribute a beta version to a number of users. Retrieve their satisfaction and adjust the Op until meet the highest satisfaction. A transit network in a city has similar properties in every part of it. Then, one Op could be assigned to one transit network.

The path between OD calculated with k-shortest path is in the integrated network, including walking path, transit path and transfer path. The transfer will be automatically divided into three steps, namely access, walking on road network and egress. During analyzing the integrated network to generate the optimal paths, if a bus stop appears as a node, the algorithm will search for all of the links connected to it, and the optimal path spanning tree will be extended. The nearest stop around the egress link in the road network and the next stop in the transit network will be considered at first and then the algorithm will continue. Next, update the links’ impedance in the current shortest path and the transit network then another shortest path will be generated. Those steps run again and again until the specified criterion is reached.

4. CASE STUDY

In order to test the validity and feasibility of the network structure and the new k-shortest path algorithm, an ideal and simple case has been built first. The road network is built on a
popular GIS platform as a gray grid (16×9) shown in Fig. 2. The impedance of each road link is set to 1000. The bold colored dot lines in the map are transit routes on the route layer. They have stops at every intersection of the overlaid road network. The impedance of every route is set to 100. All of the links on both networks are bidirectional and the two networks are merged by dummy links. The impedance of every access link is set to 200, and every egress link is set to 50. The method in section 2 has been used to combine the two networks.

Fig. 2 displays the k-shortest paths set as bold black lines. The origin node (E1) is at the top left of the map and the destination node (M7) is at the right bottom. It shows that the searched path in the combined network calculated with k-shortest path algorithm automatically considers the effect of walking, transit and transfer during the whole trip. If a user follows this path he will walk from E1 to D1 then get on a bus at D1 and go to D3 along the transit route D1-D5. At D3 he will transfer to another transit line and then at H3, H6 he will make a transfer likewise. Finally he will get off the bus at M6 and walk to M7. The total impedance of this optimal path is 4400.

Another alternative path could be generated when the links in the first one has been partially updated. And the optimal path between the OD will be E1-D1-D5-D6-E6-M6-M7. The number of transfer becomes 1 and the total impedance becomes 5700. Other alternative paths also will be calculated by avoiding similarity. User finally chooses their favorite path among...
the path set.

5 CONCLUSIONS

The topological structure and the k-shortest path algorithm proposed by this study could be easily applied on an existing transit network. This system can efficiently generate less similar paths and provide users more wide choices than other system. Because of the simplicity of the topological structure and the k-shortest path algorithm, the developer can also easily develop a rich featured user interface for displaying and setting.

The partial overlap used in this paper just help a single user make his travel plan effectively. It could also be used as an indicator of congestion of a park and ride system which contains hundreds of transfers and paths. Control the partial overlap degree will avoid the congestion on several park and ride sites.

The future work to be continued in this study is to extend the partial overlap method to a more efficient method. The access links and egress links can be put into consideration. When updating the links’ impedance in transit network different renewed rates can be used. That should be more reasonable.

REFERENCES


Kyriacos C. Mouskos and Joshua Greenfeld, (1999) A GIS-Based Multi-modal Advanced Traveler Information System, Computer-Aided Civil and Infrastructure Engineering, 14,
1999, 267–279
