INCORPORATING NETWORK IMPACT ANALYSIS INTO ROAD ALIGNMENT OPTIMIZATION

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Abstract: In existing literature on road alignment optimization, only some cost factors related to the road itself are considered. But actually a new road is not only an isolated transportation facility, but also obviously a component part of a road network. The impact of the new road on the original network is valuable and could not be ignored. In this paper network impact analysis is incorporated into the road alignment optimization model. Each road alternative can change the topology structure of the original network. As for each alternative, flow characteristics of OD traffic on the links are obtained through traffic assignment, and thus changed travel time and environmental load of OD traffic on the network can be estimated. They are converted into monetary equivalents and are regarded as the benefit incurred by the alternative. The ratio of benefit to cost is used as the objective function to evaluate each alternative. Genetic Algorithm is employed to solve this model.

Key Words: Road Alignment Optimization, Genetic Algorithm, Traffic Assignment, Environmental load

1. INTRODUCTION

Road alignment optimization problem is to find the most economical road alternative connecting two given end points based on topography, soil conditions, socioeconomic factors and environmental impacts, while satisfying a set of design and operational constraints. Due to the complexity of this problem, traditional road alignment optimization problem requires experienced engineers to repetitively evaluate various alternatives in order to determine the most promising one. Since the number of alternatives connecting two given end points is infinite, a manual method may arrive at a merely satisfactory solution rather than a near-optimal one.

Such road alignment optimization problems have attracted much research interest over the past three decades. Various mathematical methods such as dynamic programming, numerical search, linear programming and network optimization have been employed for this problem in
the earlier literature. Most methods are devoted to optimizing either the horizontal alignment or the vertical alignment. However, along with the rapid development of computer and information technology, Geographic Information System (GIS) and digital spatial data have been widely applied recently. Many new methods based on GIS have been put forward. Jong et al. (2003) have developed an evolutionary model for simultaneously optimizing three-dimensional highway alignments. The model emphasizes the application and realization of Genetic Algorithm (GA) in highway alignment optimization. Jha et al. (2003) have developed a criteria-based decision support system based on GIS for selecting highway alignments. In addition, Jha et al. (2004) have developed an alignment optimization model based on GIS and GA. In general, the characteristics of recent studies are listed as follows: (1) The models are developed based on GIS; (2) The models employ GA as a solution method. (3) The models emphasize to optimize simultaneously three-dimensional road alignment; (4) In the selecting process, a number of factors such as user costs (cost of vehicle operation, travel time cost, accident cost, etc.), supplier costs (earthwork cost, construction cost, etc.) and environmental costs are introduced in the model to judge the alternatives.

However, most models found in the literature are only based on the theory of road design. None of the existing methods have considered the impact of the new road on Level of Service (LOS) of the original road network. But actually as for the new road, it is not only an isolated transportation facility, but also obviously a component part of a road network. Thus it is valuable that the effect of new road on original road network can be considered in road alignment optimization. In this paper network impact analysis is incorporated into road alignment optimization model. The impact of the new road on the original road network is represented by two factors, one is changed travel time of OD traffic, the other one is changed environmental load of OD traffic. In addition, the existing methods have still not explained well how to realize the design of the road alignment in GIS. Undoubtedly, that is a difficult issue due to the complexity of automatic design of the road alignment. It is necessary to make the alignments of road alternatives more similar to those in the practice project. Thus the design of the alignment based on GIS is carried out in terms of design and operational requirements in this model.

The rest of this paper is organized as follows. In the following Section 2, the framework of model is described. Section 3 gives a detailed description of each component in this model. In Section 3.1 the impact of the new road on the original network is introduced. The description of road alternatives in GIS is presented in Section 3.2. The design method of horizontal and vertical alignment of the alternative is described in Section 3.3. Section 3.4 introduces the calculation method of some costs related to the road itself. Section 4 explains how to solve the model by GA. A numerical example is presented to test this model and the results are discussed in Section 5. The paper ends with conclusions in Section 6.

2. THE FRAMEWORK OF MODEL

The objective of the model presented in this paper is to find the optimal road alternative between two given end points in road network whose ratio of benefit to cost is maximal. The development of the model is based on GIS. In GIS, this model records some vector graph layers such as road network, Digital Elevation Model (DEM) and natural surface characteristics in the target region. This model is solved by GA. Fig.1 illustrates the proposed model. First, a set of road location alternatives between two given end points is generated randomly based on DEM data. Second, specific alignments of alternatives are designed
automatically in GIS. Third, as for each alignment alternative, various project costs involved it, i.e. construction cost, earthwork cost and construction cost of potential bridges or tunnels are calculated respectively in GIS. Besides the costs mentioned above, traffic accident cost represents a great economic loss, and is also regarded as the factor to evaluate the alternative. Accident costs are usually estimated by multiplying accident rates and the average cost per accident (Jong et al. 1999). Since the causes of accidents are many and complex, it is very difficult to specify the exact relation between accident rates and alignment configurations. For this reason and due to the lack of empirical data, accident costs are not considered in the cost functions. Each road alternative can change the topology structure of original road network, and new topology of the changed network will be established. Then the same OD traffic is assigned to the network of each alternative through user equilibrium traffic assignment. As a result, flow attributes on links, such as traffic volume, travel speed and time, can be obtained. Then travel time and environmental load of OD traffic are calculated. By comparing them with those of original network, changed values of travel time and environmental load of OD traffic on the network are calculated. After that, the monetary equivalents of these factors are estimated and the sum of them is regarded as the benefit of each road alternative. Total cost is obtained through summing the various costs shown in the Figure. In addition, road design criterion cannot entirely be satisfied in the process of road design based on GIS, and thus here the penalty functions are used to correct it. Finally, the fitness function consisting of the cost, the benefit and the penalty is built to evaluate the alternatives. Subsequently, a new set of alternatives are obtained through operators of selection, crossover and mutation in GA, and the process will be repeated until the convergence condition is satisfied.

Figure 1 The framework of model
3. RESEARCH METHODOLOGY

3.1 The Impact Analysis of New Road on Road Network

Building a new road in the road network is able to change the topology of the network, and then further change LOS of the network. In transport planning theory, “Braess paradox” indicates that building a new road in the road network does not necessarily improve Level of Service of road network (Sheffi, 1985). Thus, it is valuable that the impact of a new road on road network is taken into account in road alignment optimization. But in the evaluation process the impact of alignment alternatives on original network is always neglected. Therefore, planners fail to grasp the impact of the final road alternative on the network. In this paper network impact analysis is incorporated into road alignment optimization. The impact is represented by two factors, changed travel time and environmental load of OD traffic in the road network.

In road alignment optimization model, many alternatives are generated for evaluating, and each alternative is corresponding to a separate road network. In order to study the impact of the alternative on network, and estimate the travel time and environmental load of OD traffic, the topology of network should be automatically built for each alternative. The automatic topology method is proposed below.

Suppose \( \Lambda = [c_1, c_2, c_3, \ldots, c_i, \ldots, c_{n-2}, c_{n-1}, c_n] \) represents a road alternative. It can be represented in GIS by a polyline. In order to keep the topological structure of the original road network for subsequent topology operations, the polyline is tracked as a variable rather than an object in GIS. XY coordinates of the intersections of the variable and the existing links are calculated in GIS. Columns O, D of Table 1 record ID of start node and end node of existing link, respectively. And Column Dis stands for the length of the existing link. Columns MM1-MM5 record IDs of new added nodes; columns D1-D5 record the distances from a existing origin to the nodes of MM1-MM5. The advantage of this table is that it records the changes without destroying the objects and attribute table in GIS, while its disadvantage is that it cannot record the new road. Therefore, the new road is recorded separately, and then a topology table for the new network is synthesized.

Table 1 Recording method for added nodes in road network

<table>
<thead>
<tr>
<th>RoadID</th>
<th>O</th>
<th>MM1</th>
<th>MM2</th>
<th>MM3</th>
<th>MM4</th>
<th>MM5</th>
<th>D</th>
<th>Dis</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>MMM</th>
<th>C1</th>
</tr>
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<tbody>
<tr>
<td>29</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>277.245</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,600</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>27</td>
<td>579.577</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3,200</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>28</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>27</td>
<td>423.045</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3,200</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>27</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>29</td>
<td>947.082</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3,200</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>27</td>
<td>301</td>
<td>302</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>1,622.15</td>
<td>192.819</td>
<td>317.235</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3,200</td>
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<td>34</td>
<td>29</td>
<td>303</td>
<td>305</td>
<td>306</td>
<td>307</td>
<td>0</td>
<td>31</td>
<td>1,115.84</td>
<td>506.946</td>
<td>694.46</td>
<td>801.665</td>
<td>0</td>
<td>4</td>
<td>3,200</td>
<td></td>
</tr>
</tbody>
</table>

In order to estimate travel time and environmental load of OD traffic, after automatic topology, Frank-Wolfe algorithm (Lee, et al. 2003) is used to assign the same OD traffic on road network. And the traffic volume, travel time and traffic average speed of vehicles are got in the links for all alternative cases. Finally travel time and environmental load of OD traffic are calculated based on road network corresponding to each alternative. In addition, they need to be converted into monetary equivalents. The detailed formulas are listed below.

\[
C_{TT} = c_i \sum_a (x_{a}^{AB} \times t_{a}^{AB} + x_{a}^{BA} \times t_{a}^{BA})
\]  

Where, \( C_{TT} \) denotes total travel time cost of OD traffic in the peak hour; \( c_i \) denotes value of
time per hour; $x_{a}^{AB}, x_{a}^{BA}$ denote traffic flow in $AB$ and $BA$ direction of link $a$, respectively; $t_{a}^{AB}, t_{a}^{BA}$ denote travel time of vehicle in $AB$ and $BA$ direction of link (unit: hour).

Environmental load cost of OD traffic in the peak hour can be calculated by the formulas below. First, pollutants (CO, HC and NOx) emitted from vehicle are calculated. Next they are changed to monetary equivalents. It means the costs needed to cancel the negative effects of the pollutants.

$$C_{p} = C_{p}^{CO} + C_{p}^{HC} + C_{p}^{NOx}$$ (2)

$$C_{p}^{CO} = c_{CO} \sum_{a} l_{a} (\mu_{a,CO}^{AB} \times x_{a}^{AB} + \mu_{a,CO}^{BA} \times x_{a}^{BA})$$ (3)

$$C_{p}^{HC} = c_{HC} \sum_{a} l_{a} (\mu_{a,HC}^{AB} \times x_{a}^{AB} + \mu_{a,HC}^{BA} \times x_{a}^{BA})$$ (4)

$$C_{p}^{NOx} = c_{NOx} \sum_{a} l_{a} (\mu_{a,NOx}^{AB} \times x_{a}^{AB} + \mu_{a,NOx}^{BA} \times x_{a}^{BA})$$ (5)

Where, $C_{p}$ denotes environmental load cost of OD traffic; $C_{p}^{CO}, C_{p}^{HC}, C_{p}^{NOx}$ denote indices of monetary loss for pollutants CO, HC, NOX; $c_{CO}, c_{HC}, c_{NOx}$ denote indices of monetary loss for unit pollutants CO, HC, NOX; $x_{a}^{AB}, x_{a}^{BA}$ denote traffic volume in $AB$ and $BA$ direction of link $a$, respectively; $l_{a}$ denotes the length of link $a$ (unit: kilometer); $\mu_{a,CO}^{AB}, \mu_{a,HC}^{AB}, \mu_{a,HC}^{BA}, \mu_{a,NOx}^{AB}, \mu_{a,NOx}^{BA}$ denote emission factors of CO, HC, NOX in $AB$ and $BA$ direction of link $a$, respectively, which are calculated by the interpolated method from Tab. 2 (unit: g/km-pcu).

<table>
<thead>
<tr>
<th>Pollutant (g/km)</th>
<th>16</th>
<th>32</th>
<th>48</th>
<th>64</th>
<th>80</th>
<th>97</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>59.55</td>
<td>30.11</td>
<td>21.26</td>
<td>17.15</td>
<td>14.34</td>
<td>12.47</td>
</tr>
<tr>
<td>HC</td>
<td>7.08</td>
<td>4.63</td>
<td>3.63</td>
<td>2.98</td>
<td>2.5</td>
<td>2.27</td>
</tr>
<tr>
<td>NOX</td>
<td>3.16</td>
<td>3.55</td>
<td>3.9</td>
<td>4.39</td>
<td>4.81</td>
<td>5.16</td>
</tr>
<tr>
<td>Sum</td>
<td>69.79</td>
<td>38.29</td>
<td>28.79</td>
<td>24.52</td>
<td>21.65</td>
<td>19.90</td>
</tr>
</tbody>
</table>

Table 2 The emission factors of vehicles

3.2 The Description of Initial Road Alternatives

In the process of traditional road alignment optimization, the number of road alternatives for selection is limited so that the final selected road is the merely satisfactory solution rather than a near-optimal one. Therefore, a sufficient number of alternatives must be generated to ensure that the final solution is the optimal solution as close as possible. Thus this paper employs GA to generate the sufficient number of alternatives in the process of optimization. The method of representation of road alternatives in our proposed model is described below.

In original road network the start point and end point are given beforehand in GIS. As shown in Fig. 2, $n$ lattice groups are extracted between two given end points from DEM layer in GIS according to the planned road corridor. Each lattice group includes the certain number of lattices. Control lattice is randomly generated in each lattice group. The centroid of each control lattice is taken as a control point of a road alternative. By connecting these points and two given end points, an initial road alternative is generated. The road alternative is represented by the set of centroids.

Suppose that an initial alignment is composed of $n$ control points (not including two given
end points), there would be \( n \) lattice groups. After extracting these lattice groups from DEM (DEM layer consists of a number of lattices), IDs of lattices of each lattice group are represented by the serial integers. The road alternative can be expressed by a numerical string in which each digit is corresponding to ID of a selected lattice in a lattice group. The initial road alternatives can be generated with formula (6) which means a control lattice is randomly selected in each lattice group. As shown in Fig. 2, by connecting all these centroids of control lattices marked by pentacles, the initial alternative \( k \) of new road is generated.

\[
\Lambda_k = [c_1, c_2, c_3, \ldots, c_i, \ldots, c_{n-2}, c_{n-1}, c_n]_k, \quad 1 \leq i \leq n, \quad c_i = \text{Random}(c_i^{\text{min}}, c_i^{\text{max}}) \quad (6)
\]

Where, \( \Lambda_k \) denotes a road alternative \( k \); \( n \) is the number of control points; \( c_i^{\text{min}}, c_i^{\text{max}} \) are smallest and largest ID of lattices in the \( i \)th lattice group; \( c_i \) is ID of the certain lattice generated randomly in the \( i \)th lattice group, and it is an integer between \( c_i^{\text{min}} \) and \( c_i^{\text{max}} \).

3.3 The Design of Road Alignment based on GIS and DEM

As the calculations of some costs in the objective function are dependent on the alignment of the road alternative, it is necessary to design the horizontal and vertical alignments of road alternatives as explicitly as possible. Albeit it is not possible at the current stage to design an alignment that satisfies completely a set of design and operational constraints, it is imperative and achievable to work out a similar design. In the projects the design of the road alignment is implemented through manual operations by road engineers, but this solution algorithm cannot be interrupted by manual operations. Thus the automatic design method of the alignment is developed. The sections below will discuss the design methods of horizontal and vertical curves, and its automatic realization in GIS.

Section 3.2 describes that an initial road alternative is represented by a set of centroids and two given end points. Linking each pair of successive points with a straight-line section will generate a piecewise linear trajectory. As shown in the Fig.3, an initial road alternative is represented by a polyline object in GIS. Next, circular curves must be fitted to connect the tangent sections at the control points with nonzero intersection angles. An iterative computer algorithm for doing this is developed in GIS. In order to keep the resulting alignment continuous for each point, the radius at points with large intersection angles sometimes may be less than the minimum required by road design constraints. In such a case, the alignment must be penalized during the evaluation process (see Section 4).
generated by the algorithm is smooth and composed of tangent sections and circular curves. Although the spiral transition curves in the real design alignment are omitted here, the alignment is still suitable enough to evaluate.

Fig. 3 shows that a horizontal alignment is described by a set of stake points in GIS. These stake points include the equidistant points each of which is created at regular intervals from the given start point, and some special points involving points of tangency (LCP<sub>i</sub>, CLP<sub>i</sub>), points of curvature, and the middle points of circular curves (MCP<sub>i-1</sub>, MCP<sub>i</sub>, MCP<sub>i+1</sub>). The purpose of the algorithm for the horizontal alignment design is to determine XY coordinates of each stake point. CP<sub>i</sub>, CP<sub>i-1</sub>, CP<sub>i+1</sub> denote three successive control points. In order to insert a circular curve at CP<sub>i</sub>, its length of tangent denoted by \( T_i^H \) should be firstly determined and then other variables can be calculated. In general the length of tangent at each control point is determined by road engineers. Here the equation (7) is developed for determining this important variable so that the continuous loop of calculations is guaranteed without interrupted inputs.

\[
T_i^H = \gamma_1 \times \min(L_{i-1}^H, L_{i+1}^H)
\]  

(7)

Where, \( L_{i-1}^H, L_{i+1}^H \) denote the horizontal distances of CP<sub>i-1</sub>CP<sub>i</sub> and CP<sub>i+1</sub>CP<sub>i</sub> respectively. \( \gamma_1 \) denotes the parameter specified by the user and is the real number between 0 and 1.

Let \( x_j, y_j, d_j \) denote XY coordinates of the jth stake point and its distance from the given start point respectively. There variables can be calculated according to the design theory of road horizontal alignment. Let \( z_j^0 \) denote the ground elevation of the jth stake point. It is obtained according to its XY coordinates in DEM (Each lattice in DEM layer has one elevation value).
After the horizontal alignment of the road alternative is designed, its vertical alignment is designed next. As shown in Fig. 4, firstly, the designed horizontal alignment is straightened to the straight line. And then linking each pair of successive stake points with a straight-line section in the vertical profile will generate actual ground line. In the design of vertical alignment, the middle points of circular curves such as MCP\textsubscript{i} are taken as the control points in the design of vertical alignment. A quadratic parabola must be fitted to connect the tangent sections at point MCP\textsubscript{i} with nonzero intersection angles. An iterative computer algorithm for the design of the vertical alignment is developed. In addition, this cannot ensure that all the vertical grades meet the design criteria. In such cases, the alignment must be penalized during the evaluation process (See Section 4).

\begin{equation}
T_{i}^{V} = \gamma_{2} \times \min(L_{i,i-1}^{V}, L_{i+1,i}^{V})
\end{equation}

Where, \(T_{i}^{V}\) denotes the length of tangent of the parabola inserted at the control point MCP\textsubscript{i}; \(\gamma_{2}\) denotes the parameter specified by the user and its value is less than \(\gamma_{1}\) and between 0 and 1; \(L_{i,i-1}^{V}, L_{i+1,i}^{V}\) denote the horizontal distances between MCP\textsubscript{i} and MCP\textsubscript{i-1}, and between MCP\textsubscript{i+1} and MCP\textsubscript{i}, respectively.

After the horizontal and vertical alignments are designed, we can obtain the center line of road alternative. In order to calculate the earthwork cost, the cross section of road alternative is realized easily by the buffer function in GIS.

3.4 Costs Relevant to Road Itself

In the evaluation process of alternatives, various costs should be considered. But due to their
complexity not all can be formulated. In this paper, several road costs such as road cost of construction, earthwork, bridges and tunnels are considered. Calculation methods of those costs are described as follows:

Construction cost is calculated by the following equation (9).

\[ C_B = c_B \sum_{j=1}^{m-1} \sqrt{(x_{j+1} - x_j)^2 + (y_{j+1} - y_j)^2 + (z_{j+1} - z_j)^2} \]  

(9)

Here, \( C_B \) is basic construction cost; \( c_B \) is average construction cost per unit length; \( m \) is the number of all stake points along the alignment; \( x_{j+1}, y_{j+1}, z_{j+1}, x_j, y_j, z_j \) are XYZ coordinates of the \( j+1 \)th, \( j \)th stake points, respectively.

For bridge cost, firstly the length of new road crossing the river is calculated by the overlay function in GIS. Small bridge, medium bridge and large bridge are classified according to their length, and then the costs of bridges being built are estimated.

\[ C_{Br} = \sum_k c_{Br} \times l_k \]  

(10)

Here, \( C_{Br} \) is the cost of bridge; \( l_k \) is the length of bridge; \( c_{Br} \) denotes the construction cost per unit length for small, medium and large bridge.

If the cutting depth is over 20m, a tunnel is supposed to be built. And the cost is estimated by the equation given below.

\[ C_{Tu} = c_{Tu} \sum_{j=1}^{s} (d_{j+1} - d_j), s = \{ j \, | \, z_j^0 - z_j \geq 20 \} \]  

(11)

Here, \( C_{Tu} \) is the construction cost of tunnel; \( c_{Tu} \) is the construction cost per unit length; \( s \) denotes the set of stake points where the tunnel is built.

Earthwork cost includes cutting cost and filling cost. Two costs are calculated separately. In calculation process, each pair of two successive stake points is taken as the calculated unit. Thus, there are perhaps three situations in each unit, such as only cutting project, only filling project and the project of cutting and filling. In the section of only cutting, its cost is calculated by the equation (12). The equation (13) is used in the section of only filling. The section which involves cutting and filling is calculated by the equation (14) or (15). The total earthwork cost represented by \( C_E \) is calculated by the equation (16).

\[ C_{j+1,j}^{fe} = \frac{1}{2} \left[ (z_{j+1}^0 - z_{j+1}) + (z_j^0 - z_j) \right] \times (d_{j+1} - d_j) \times w \times c_{fe} \]  

(12)

\[ C_{j+1,j}^{de} = \frac{1}{2} \left[ (z_{j+1}^0 - z_{j+1}) + (z_j^0 - z_j) \right] \times (d_{j+1} - d_j) \times w \times c_{de} \]  

(13)

\[
\begin{cases}
C_{j+1,j}^{de} = \frac{1}{2} \times \frac{(z_{j+1}^0 - z_{j+1})^2}{(z_{j+1}^0 - z_{j+1}) + (z_j^0 - z_j)} \times (d_{j+1} - d_j) \times w \times c_{de} \\
C_{j+1,j}^{fe} = \frac{1}{2} \times \frac{(z_j^0 - z_j)^2}{(z_j^0 - z_j) + (z_j^0 - z_j)} \times (d_{j+1} - d_j) \times w \times c_{de}
\end{cases}
\]  

(14)

\[
\begin{cases}
C_{j+1,j}^{fe} = \frac{1}{2} \times \frac{(z_{j+1}^0 - z_{j+1})^2}{(z_{j+1}^0 - z_{j+1}) + (z_j^0 - z_j)} \times (d_{j+1} - d_j) \times w \times c_{de} \\
C_{j+1,j}^{de} = \frac{1}{2} \times \frac{(z_j^0 - z_j)^2}{(z_j^0 - z_j) + (z_j^0 - z_j)} \times (d_{j+1} - d_j) \times w \times c_{de}
\end{cases}
\]  

(15)
Here, $z_{j+1}, z_{j+1}, z^0_j, z_j$ denote the ground height and designed height for the $(j+1)$th, $j$th stake point, respectively; $d_{j+1}, d_j$ denote the cumulative distance for the $(j+1)$th, $j$th stake point, respectively. $w$ denotes the design width of new road; $c^e_{je}, c^0_{de}$ denote cutting and filling cost of unit volume, respectively; $C_{j+1}, C^e_{j+1}, C^d_{j+1}$ denote filling, cutting cost of the $(j+1)$th, $j$th stake point. $C^e_{je}, C^d_{de}$, denote filling and cutting cost of road alternative, respectively.

4. THE APPLICATION OF GENETIC ALGORITHM IN THIS MODEL

It is proved in the literature (Jong et al. 2003; Jha et al. 2004) that GA is a powerful tool to solve road alignment optimization problem. Here GA is employed for this optimal search. In GA, this optimization problem is treated as the environment, and each set of possible alternatives is treated as the population of each generation. Each alternative in the population is encoded into a string representation called a chromosome. The IDs of control cell are treated as decision variables.

The fitness value of each individual is calculated by the following equation (17).

$$ F = \begin{cases} B/C + \gamma / P, & if \, B > 0 \\ 0 / C + \gamma / P, & if \, B < 0 \end{cases} $$

(17)

Where $B$ is the benefit of an alternative; $C$ is the cost of an alternative; $\gamma$ is the parameter and the positive number greater than 0; $P$ is the penalty cost related to design constraints.

$$ B = [C^0_{TT} - C^0_{TT}) + (C^0_p - C^0_p)] \times 2 \times 365 \times u $$

(18)

Here, $C^0_{TT}$, $C^0_p$ denote travel time and environmental load value of OD traffic in original network respectively; $u$ denotes the design life span of the new road; 2 means the morning and evening peak hour; 365 means the number of days in a year.

$$ C = C_R + C_{Be} + C_{Ta} + C_E $$

(19)

Here, $C$ denotes the total cost related to road itself.

Since not all alignments of alternatives can meet the design constraints. The most important constraints in road design are minimum radius and maximum gradient. Referring to penalty functions in Jha et al. (2004), the penalty functions for alternatives are introduced to ensure
the effective evaluation. The penalty cost $P$ is divided into two parts as follows:

$$P_r = \sum_{i=1}^{n} \left[ \alpha_0 + \alpha_i \times (R_{\text{min}} - r_i)^{\alpha_2} \right] \text{ if, } r_i < R_{\text{min}}$$

$$P_g = \sum_{i=1}^{n} \left[ \beta_0 + \beta_i \times (|g_i| - G_{\text{max}})^{\beta_2} \right] \text{ if, } |g_i| > G_{\text{max}}$$

Here, $P_r$ denotes the penalty function for violating minimum radius constraint; $P_g$ denotes the penalty function for violating maximum gradient constraint; $r_i$ is the radius of circular curve of the $i$th control point; $R_{\text{min}}$ denotes the minimum radius permitted in the design constraint; $g_i$ is the gradient of the $i$th control point of the vertical curves; $G_{\text{max}}$ denotes the maximum gradient permitted; $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$ are user-specified parameters.

The procedure of GA in this model is listed as follows:

**Step 0:** (Initialization) Initial alternatives are generated randomly by the method presented in Section 3.2. The number of alternatives is $P_{size}$.

**Step 1:** (Calculating the fitness value of each individual) Evaluate the designed alternatives in the current generation based on their fitness values, and then identify the best alternative and the poorest one. And move to the next Step 2.

**Step 1.1:** According to Section 3.2, the horizontal and vertical alignment of each alternative is designed in GIS.

**Step 1.2:** The costs related to road itself are calculated by the method in Section 3.4.

**Step 1.3:** Travel time and environmental load of OD traffic are estimated by the method in Section 3.1. Comparing with corresponding indices in original network, the benefit is calculated.

**Step 1.4:** Based on the above result, the fitness of each alternative is calculated by the equation (31).

**Step 2:** (Generate the population of next generation).

**Step 2.1:** (Selection) According the fitness values of alternatives in previous generation, the alternatives for reproducing offspring are selected by the method of roulette wheel.

**Step 2.2:** (Crossover) Here employs arithmetic crossover operator. Based on crossover probability $P_c$, two new offspring are reproduced from two parents’ chromosomes by the equation (22).

$$\Lambda'_m = \omega\Lambda'^{-1}_m + (1 - \omega)\Lambda'^{-1}_n$$

$$\Lambda'_n = \omega\Lambda'^{-1}_n + (1 - \omega)\Lambda'^{-1}_m$$

Here, $\Lambda'^{-1}_m, \Lambda'^{-1}_n$ denote two parents. $\Lambda'_m, \Lambda'_n$ denote two offspring. $\omega$ denotes the random number between 0 and 1.

**Step 2.3:** (Mutation) Here employs the non-uniform mutation operator. Based on mutation
probability $P_m$, the gene is mutated to reproduce the new offspring. Supposed that the chromosome $N = [c_i', c_2', c_3', \ldots, c_{n-2}', c_{n-1}', c_n']$ is mutated at the $k$th ($k = \text{Random}(0,n)$) gene represented by $c_i' \ (c_i' \in [c_i^{\min}, c_i^{\max}])$. New offspring represented by $\tilde{N} = [c_1', c_2', c_3', \ldots, c_{n-2}', c_{n-1}', c_n']$ is reproduced by the equation (23) below.

$$
\tilde{c}_k' = \begin{cases} 
    c_k' + \Delta(t, c_k^{\max} - c_k') & \text{if, } \text{Random}(0,1) = 0 \\
    c_k' + \Delta(t, c_k' - c_k^{\min}) & \text{if, } \text{Random}(0,1) = 1 
\end{cases} 
$$

The function $\Delta(t, y)$ returns a random value in the range $[0, y]$ such that $\Delta(t, y)$ approaching 0 as $t$ increases. This property enables the operator to search the search space uniformly at initial generations and very locally at later generations. The function $\Delta(t, y)$ is expressed as

$$
\Delta(t, y) = y \times (1 - r^{(1-t/T_{max})})^\lambda 
$$

Where $r$ is a random number between $[0, 1]$; $t$ is the current generation number; $T_{max}$ is the maximal generation number; $\lambda$ is a user specified parameter which determines the degree of non-uniformity.

**Step 2.4:** (Elitist selection strategy) Calculate the fitnesses of new population in the current generation. Then the alternative whose fitness value is the lowest in this generation is replaced with the one whose fitness value is the highest in the previous generation.

**Step 3:** (Test of termination) The process is repeated by returning step 2 until the termination condition has been reached.

5. NUMERICAL EXAMPLE

In this section, a numerical example is employed to test the effectiveness of the proposed model and evolutionary program. A region with 35 traffic analysis zones, 433 existing links, 287 nodes and 40,819 DEM cells is used as the study area. Node 27 and 146 are supposed to two given points of the new road. The parameters of the GAs are set as $p_c = 0.6$, $p_m = 0.001$, $P_{scx} = 60$, $T_{max} = 100$, $\lambda = 3$. Suppose that the design speed of new road is 80km/h and the width of the road is 24m, the minimum radius of circular curve is 1000m, the maximum vertical grade is 4% and the interval distance between stations is 50m. The unit costs are set as 5$/m^3$ for cutting, 1.5$/m^3$ for filling, 12million $$/km$ for basic construction cost. Value of time is 0.1$/min$. The life span of the road is 30 years. Due to the lack of real data, some parameters can not be estimated. Thus suppose that $\gamma_1 = 1/2, \gamma_2 = 1/3$, $\alpha_0 = 1, \alpha_1 = 0.0001, \alpha_2 = 1$, $\beta_0 = 1, \beta_1 = 10000, \beta_2 = 2$, $\gamma = 100$ are given. From existing studies many indices of monetary loss for unit pollutants can be found, here the values ($c_{CO} = 11.62$/kg, $c_{HC} = 968.75$ $$/kg$, $c_{NOx} = \text{387.5}$$$/kg$) proposed by Nakamura et al. (1997) are adopted.

MapInfo system is used as the spatial database and MapBasic is used as the basic computing language for the algorithm. Although with MapBasic data and spatial analyses in MapInfo can be used effectively, its calculating speed is far from the satisfaction. Thus, DLL technique of
C++ is used to program the algorithm of traffic assignment and design of road alignment. This program cost nearly 24 hours to run one time.

Table 3 The result in GA

<table>
<thead>
<tr>
<th>Generation</th>
<th>Fitness value</th>
<th>Benefit (million$)</th>
<th>Cost (million$)</th>
<th>Penalty cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>363.80</td>
<td>9344.03</td>
<td>26.41</td>
<td>9.98</td>
</tr>
<tr>
<td>1</td>
<td>366.12</td>
<td>9100.00</td>
<td>25.39</td>
<td>13.02</td>
</tr>
<tr>
<td>9</td>
<td>430.73</td>
<td>9727.86</td>
<td>23.09</td>
<td>10.70</td>
</tr>
<tr>
<td>20</td>
<td>464.72</td>
<td>10708.82</td>
<td>23.70</td>
<td>7.72</td>
</tr>
<tr>
<td>27</td>
<td>480.49</td>
<td>10581.54</td>
<td>22.63</td>
<td>7.72</td>
</tr>
<tr>
<td>36</td>
<td>483.51</td>
<td>10592.16</td>
<td>22.51</td>
<td>7.72</td>
</tr>
<tr>
<td>42</td>
<td>484.22</td>
<td>10725.69</td>
<td>22.76</td>
<td>7.72</td>
</tr>
<tr>
<td>55</td>
<td>486.09</td>
<td>10698.75</td>
<td>22.61</td>
<td>7.72</td>
</tr>
<tr>
<td>79</td>
<td>487.25</td>
<td>10736.30</td>
<td>22.64</td>
<td>7.72</td>
</tr>
</tbody>
</table>

Table 4 The result of some factors in road alternatives

<table>
<thead>
<tr>
<th>Generation</th>
<th>Factors based on network</th>
<th>Factors related to road itself</th>
<th>Penalty cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{t_{n}}$ (h)</td>
<td>HC (kg)</td>
<td>CO (kg)</td>
</tr>
<tr>
<td>0</td>
<td>59740.53</td>
<td>5076.17</td>
<td>36738.94</td>
</tr>
<tr>
<td>1</td>
<td>68372.42</td>
<td>5069.47</td>
<td>36782.65</td>
</tr>
<tr>
<td>9</td>
<td>70525.83</td>
<td>5036.42</td>
<td>36301.35</td>
</tr>
<tr>
<td>20</td>
<td>60595.76</td>
<td>5016.57</td>
<td>36235.62</td>
</tr>
<tr>
<td>27</td>
<td>60722.03</td>
<td>5022.85</td>
<td>36297.64</td>
</tr>
<tr>
<td>36</td>
<td>60720.27</td>
<td>5022.56</td>
<td>36295.91</td>
</tr>
<tr>
<td>42</td>
<td>60588.60</td>
<td>5015.65</td>
<td>36228.60</td>
</tr>
<tr>
<td>55</td>
<td>60601.08</td>
<td>5017.21</td>
<td>36240.62</td>
</tr>
<tr>
<td>79</td>
<td>60586.84</td>
<td>5015.36</td>
<td>36226.87</td>
</tr>
</tbody>
</table>

Fig. 5 shows the change trend of average fitness value of each generation during GA evolutions. It can be seen that average fitness value keeps nearly constant from 79th
generation and the optimal result is obtained. Table 3 and Table 4 show some important factors from the best alignments of some selected generations respectively. Travel time of OD traffic is 96186.48 h in original road network. Emission volumes of OD traffic such as HC, CO and NOX are 5448.30kg, 40283.689kg and 4504.61kg, respectively. The row represented by 79th generation records some factors of the best alignment. Travel time, HC and CO decrease 37%, 8% and 10% from original, respectively. Only NOX increases 1%. Fig.6 and Fig.7 show the final road alignment in 2-D and 3-D map. It is found that the final alignment prefers passing through a flat area for avoiding more earthwork cost. And the final alignment generally tends to intersect with original network at original nodes and does not usually break the original links.

![Figure 6 The final road alignment in road network](image1)

![Figure 7 The final road alignment in 3-D map](image2)

It is noted that the final alternative shown in the example still does not satisfy the constraint of
minimize radius of circular curve. Jong et al. (2003) mentioned “The penalty method does not guarantee that constraints are always satisfied. If constraint violations exist in the final solution, they are usually very slight and can be easily avoided by setting the constraint bounds slightly more tightly than necessary.”. However, the method to make the solution always feasible should be considered further. In GA, the alternatives which violated the constraints should be removed and replaced with the new alternatives which are regenerated by GA operators and satisfy the constraints.

6. CONCLUSIONS

This paper develops a model to optimize road alignment based on road network. The change of travel time and environmental load of OD traffic incurred by new road is considered as the important factors in the evaluation process. GA is employed to solve the model. The applications of GA effectively increase the diversity and rationality of road alternatives. Based on GIS, the model has an easier access to land use data and DEM data of the study area, it can facilitate the calculation of relevant costs to a great extent. A numerical example has verified the feasibility and effectiveness of this method.

Although road alignment designed in GIS can still not meet completely the constraints of design like as that designed in AutoCAD, it is particularly suitable for initial screening of road alignments. Due to the lack of real data, some parameters in the model could not be estimated and only be supposed for making the calculation easy. But the model still realizes our expected results. It is noted that travel time and environmental load of OD traffic are not visible indicators; these factors based on road network may not be of great concern from the supplier’s viewpoint. But they are more valuable to be considered for urban planners and road engineers in road alignment optimization.

REFERENCES